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A free boundary problem modeling the invasion of species

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1 Introduction

The spreading (migration) of an invasive or new species is one of the most important topics in mathematical ecology. A lot of mathematicians have made efforts to develop various invasion models and investigated them from a viewpoint of mathematical ecology. For example, invasion problem was first studied mathematically by Skellam [12]. After Fisher's work [5], travelling wave solutions for reaction-diffusion equations have been much used to model the successful invasions. See [11] for more detailed information.

Recently, Du and Lin [4] have proposed a new mathematical model to understand the spreading of an invasive or new species. Their model is described as a free boundary problem for a logistic diffusion equation:

$$\begin{cases} u_t - du_{xx} = u(a - bu), & t > 0,\ 0 < x < h(t), \\ u_x(t,0) = 0,\ u(t,h(t)) = 0, & t > 0, \\ h'(t) = -\mu u_x(t,h(t)), & t > 0, \\ h(0) = h_0,\ u(0,x) = u_0(x), & 0 \le x \le h_0, \end{cases} \tag{1.1}$$

where $\mu,\ h_0,\ d,\ a$ and b are given positive numbers and u_0 is a given nonnegative initial function. In (1.1), $u = u(t,x)$ represents a population density of an invasive or new species in one dimensional habitat. A free boundary $x = h(t)$ is a spreading front of the species, while $x = 0$ is the fixed boundary where no-flux boundary condition is imposed. The dynamics of the free boundary is determined by Stefan-like condition $h'(t) = -\mu u_x(t,h(t))$. This condition means that the population pressure at the free boundary is a driving force of the free boundary.

They derived various results about the asymptotic behavior of solutions for (1.1) as $t \to \infty$. One of very remarkable results is a spreading-vanishing dichotomy of the species; any solution (u,h) of (1.1) satisfies one of the following properties:

(a) $h(t) \to \infty$ and $u(t,x) \to a/b$ as $t \to \infty$ (called spreading of species);

(b) $h(t) \to h_\infty \le (\pi/2)\sqrt{d/a}$ and $u(t,x) \to 0$ as $t \to \infty$ (called vanishing of species).

When the spreading occurs, it is also proved that the spreading speed approaches to a positive constant k_0, i.e., $h(t) = (k_0 + o(1))t$ as $t \to \infty$. See also the paper of Du and Guo [3], where a free boundary problem similar to (1.1) is studied in higher space dimension and the same spreading-vanishing dichotomy is established. Moreover, free boundary problems for two species model are considered by [8] and [10].

Stimulated by the work of Du and Lin, we will study a free boundary problem for a reaction-diffusion equation with general nonlinearity and find the underlying principle to determine spreading or vanishing of species. Our free boundary problem is given by

$$
\text{(FBP)} \qquad \begin{cases} u_t - du_{xx} = uf(u), & t > 0,\ 0 < x < h(t), \\ u(t,0) = 0,\ u(t,h(t)) = 0, & t > 0, \\ h'(t) = -\mu u_x(t,h(t)), & t > 0, \\ h(0) = h_0,\ u(0,x) = u_0(x), & 0 \le x \le h_0, \end{cases}
$$

where μ, h_0 and d are positive constants and f is a locally Lipschitz continuous function satisfying

$$
f(u) < 0 \quad \text{for} \quad u > K \tag{1.2}
$$

with a positive constant K. Initial data (u_0, h_0) satisfies

$$
u_0 \in C^2[0, h_0] \tag{1.3}
$$

with

$$
u_0(0) = u_0(h_0) = 0 \quad \text{and} \quad u_0 \ge 0 \ \ (\not\equiv 0) \quad \text{in} \ \ (0, h_0). \tag{1.4}
$$

Differently from (1.1), we put zero Dirichlet boundary condition at the fixed boundary. This condition means that the habitat is restricted by a hostile environment from the left and that the species cannot survive on the fixed boundary.

Here it should be noted that the proof of the dichotomy theorem of Du and Lin [4, Theorem 3.3] basically depends on the logistic nonlinearity. Therefore, we have to develop new methods and ideas which enable us to study spreading and vanishing properties in general situation.

In this article, we present recent results obtained by our work [7]; so the proofs are shown in [7].

The main purposes of our work are as follows:

(i) Study global existence and uniqueness of solutions for a free boundary problem with general nonlinear term;

(ii) Construct useful tools to analyze the asymptotic behaviors of solutions for (FBP);

(iii) Make clear the mechanism of the asymptotic behaviors;

(iv) Make use of the results to get a better understanding for the invasion phenomenon.

Before giving the main results, we define spreading and vanishing of the species. We can prove that the free boundary is strictly increasing with respect to t and there exists $h_\infty := \lim_{t\to\infty} h(t) \in (0,\infty]$.

Definition 1. Let (u,h) be the solution of (FBP).

(I) *Spreading of species* is the case when

$$h_\infty = \infty \quad \text{and} \quad \liminf_{t\to\infty} u(t,x) > 0 \ \text{ for } \ 0 < x < \infty;$$

(II) *Vanishing of species* is the case when

$$\text{(i)} \quad h_\infty < \infty \quad \text{and} \quad \lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0$$

or

$$\text{(ii)} \quad h_\infty = \infty \quad \text{and} \quad \lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0.$$

2 Main Results

2.1 Fundamental Properties

We begin with the following global existence theorem for (FBP).

Theorem 1. *The free boundary problem* (FBP) *has a unique solution* (u,h) *satisfying*

$$0 < u(t,x) \le C_1, \quad 0 < h'(t) \le \mu C_2 \quad \textit{for} \ \ 0 < x < h(t), \ t \ge 0,$$

where C_1 *(resp.* C_2*) is a positive constant depending only on* $\|u_0\|_{C[0,h_0]}$ *(resp.* $|u_0\|_{C^1[0,h_0]}$*).*

We will show basic tools to investigate asymptotic behaviors of solutions for (FBP). The following is a comparison principle.

Theorem 2. *Let* $\overline{h} \in C^1[0,T]$ *and* $\overline{u} \in C(\overline{\Omega}_1) \cap C^{1,2}(\Omega_1)$ *with* $\Omega_1 = \{(t,x) \in \mathbb{R}^2 \mid 0 \le x \le \overline{h}(t) \ \textit{for}\ 0 < t \le T\}$ *satisfy*

$$\begin{cases} \overline{u}_t - d\overline{u}_{xx} \ge \overline{u} f(\overline{u}), & (t,x) \in \Omega_1, \\ \overline{u}(t,0) \ge 0, \ \overline{u}(t,\overline{h}(t)) = 0, & t \in (0,T], \\ \overline{h}'(t) \ge -\mu \overline{u}_x(t,\overline{h}(t)), & t \in (0,T]. \end{cases} \tag{2.1}$$

Moreover, let $\underline{h} \in C^1[0,T]$ *and* $\underline{u} \in C(\overline{\Omega}_2) \cap C^{1,2}(\Omega_2)$ *with* $\Omega_2 = \{(t,x) \in \mathbb{R}^2 \mid 0 \le x \le \underline{h}(t) \ \textit{for}\ 0 < t \le T\}$ *satisfy* (2.1) *with "* $\ge$ *" and "*Ω_1*" replaced by "* $\le$ *" and "*Ω_2*", respectively. If* $\underline{h}(0) \le \overline{h}(0)$ *and* $\underline{u}(0,x) \le \overline{u}(0,x)$ *in* $[0,\underline{h}(0)]$*, then it holds that*

$$\underline{h}(t) \le \overline{h}(t) \ \textit{in}\ [0,T] \quad \textit{and} \quad \underline{u}(t,x) \le \overline{u}(t,x) \ \textit{in}\ \overline{\Omega}_2.$$

Remark 1. *When* $(\overline{u}, \overline{h})$ *satisfies* (2.1), $\overline{h}(0) \geq h_0$ *and* $\overline{u}(0,x) \geq u_0(x)$ *in* $[0, h_0]$, *such a pair is called an upper solution of* (FBP) *for* $0 \leq t \leq T$. *A lower solution of* (FBP) *is defined in a similar manner.*

We can derive an important energy identity for any global solution.

Theorem 3. *Let* (u, h) *be any solution of* (FBP). *Then the following identity holds true:*

$$\frac{d}{2}\|u_x(t,\cdot)\|^2_{L^2(0,h(t))} + \int_0^t \|u_t(s,\cdot)\|^2_{L^2(0,h(s))}\, ds + \frac{d}{2\mu^2}\int_0^t h'(s)^3\, ds$$
$$= \frac{d}{2}\|u_0'\|^2_{L^2(0,h_0)} + \int_0^{h(t)} F(u(t,x))\, dx - \int_0^{h_0} F(u_0(x))\, dx,$$

where $F(u) = \int_0^u sf(s)\, ds$.

Remark 2. *Theorems 1 and 3 also hold true if zero Dirichlet boundary condition on the fixed boundary in* (FBP) *is replaced by zero Neumann boundary condition.*

2.2 Asymptotic Behaviors of Solutions for (FBP)

By Theorem 1, $h'(t) > 0$; so that $h_\infty := \lim_{t\to\infty} h(t)$ exists with $h_\infty \in (0, \infty]$. The following theorem implies the vanishing property.

Theorem 4. *Let* (u, h) *be any solution of* (FBP). *If* $h_\infty < \infty$, *then*

$$\lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0.$$

In the following theorem, we will give a sufficient condition of spreading.

Theorem 5. *Suppose that* $q(x)$ *is a positive solution of*

$$\begin{cases} dq'' + qf(q) = 0, & 0 < x < h_0, \\ q(0) = q(h_0) = 0, & q(x) > 0, \quad 0 < x < h_0. \end{cases} \tag{2.2}$$

Let (u, h) *be the solution of* (FBP) *with initial data* (q, h_0). *Then the following properties hold true:*

(i) $h_\infty = \infty$;

(ii) $u_t(t,x) \geq 0$ *for* $0 < x < h(t)$ *and* $t > 0$;

(iii) $\lim_{t\to\infty} u(t,x) = v^*(x)$ *uniformly in any compact subset of* $[0, \infty)$,

where v^* *is a minimal positive solution of*

$$\begin{cases} dv'' + vf(v) = 0, & x > 0, \\ v(0) = 0, & v(x) > 0, \quad x > 0 \end{cases} \tag{2.3}$$

satisfying $v^*(x) \geq q(x)$ *in* $[0, h_0]$.

We can prove the following spreading property as an immediate consequence of Theorems 2 and 5.

Corollary 1. *Let $q(x)$ be a positive solution of* (2.2) *and let (u,h) be the solution of* (FBP) *with initial data (u_0,h_0). If $u_0(x) \geq q(x)$ in $[0,h_0]$, then*

$$h_\infty = \infty \quad \text{and} \quad \liminf_{t\to\infty} u(t,x) \geq v^*(x) \quad \text{for } x \geq 0,$$

where v^* is a minimal positive solution of (2.3) *satisfying $v^*(x) \geq q(x)$ in $[0,h_0]$.*

Here is an estimate for the asymptotic speed of the free boundary in the spreading case. From Theorem 1, we can prove the following.

Proposition 1. *Let (u,h) be any solution for* (FBP). *If $h_\infty = \infty$, then there exists a constant C such that*

$$\limsup_{t\to\infty} \frac{h(t)}{t} \leq \mu C.$$

2.3 Application I : Asymptotic Behaviors in the Logistic Type

As an application of the main results, we will consider the following free boundary problem with a logistic reaction term:

$$\begin{cases} u_t = du_{xx} + u(a-bu), & t>0,\ 0<x<h(t), \\ u(t,0) = u(t,h(t)) = 0, & t>0, \\ h'(t) = -\mu u_x(t,h(t)), & t>0, \\ h(0)=h_0,\ u(0,x)=u_0(x), & 0\leq x\leq h_0. \end{cases} \tag{2.4}$$

Here μ, h_0, d, a and b are given positive constants and (u_0,h_0) satisfies (1.3) and (1.4).

The purpose is to study asymptotic behavior of the solution of (2.4) in more detail. We will prepare some results on an auxiliary fixed boundary problem with logistic nonlinearity:

$$\begin{cases} u_t = du_{xx} + u(a-bu), & t>0,\ 0<x<l, \\ u(t,0) = u(t,l) = 0, & t>0, \\ u(0,x) = \varphi(x), & 0\leq x\leq l, \end{cases} \tag{2.5}$$

where l is a positive number and φ is a nonnegative continuous function such that $\varphi \not\equiv 0$. Since this is a gradient system, any solution $u(t,x)$ of (2.5) converges to a solution of the corresponding stationary problem as $t \longrightarrow \infty$ (see [1] and [6]):

$$\begin{cases} dq'' + q(a-bq) = 0, & 0<x<l, \\ q(0)=q(l)=0, \\ q(x)\geq 0, \quad 0<x<l. \end{cases} \tag{2.6}$$

To be more precise, we have the following result.

Proposition 2. *Let $u = u(t, x)$ be any solution of* (2.5).

(I) *If $a \leq d(\pi/l)^2$, then $q \equiv 0$ is a unique solution of* (2.6) *and $\lim_{t\to\infty} u(t,x) = 0$ uniformly in $[0, l]$;*

(II) *If $a > d(\pi/l)^2$, then* (2.6) *has a unique positive solution $q = q_l(x)$ and $\lim_{t\to\infty} u(t,x) = q_l(x)$ uniformly in $[0, l]$.*

For the proof, see, e.g., Cantrell and Cosner [2, Corollary 3.4].

We are now ready to study spreading and vanishing properties. The following theorem implies the vanishing.

Theorem 6. *Let (u, h) be any solution of* (2.4). *If $h_\infty < \infty$, then*

$$h_\infty \leq \pi\sqrt{\frac{d}{a}} \quad \text{and} \quad \lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0.$$

The spreading in the logistic case is given by the following theorem.

Theorem 7. *Let (u, h) be any solution for* (2.4). *If $h_\infty = \infty$, then $\lim_{t\to\infty} u(t,x) = v_L(x)$ uniformly in any compact subset of $[0, \infty)$, where v_L is a unique positive solution of*

$$\begin{cases} dv'' + v(a - bv) = 0, & x > 0, \\ v(0) = 0. \end{cases} \tag{2.7}$$

Combining Theorems 6 and 7, we have the following dichotomy theorem in the logistic case.

Theorem 8. *Let (u, h) be any solution of* (2.4). *Then, either* (I) *or* (II) *holds true:*

(I) *Spreading : $h_\infty = \infty$ and $\lim_{t\to\infty} u(t,x) = v_L(x)$ uniformly in any compact subset of $[0, \infty)$, where v_L is a positive solution of* (2.7);

(II) *Vanishing : $h_\infty \leq \pi\sqrt{d/a}$ and $\lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0$.*

In the latter part, we will prove that both spreading and vanishing can happen depending on initial data.

Proposition 3. *If $h_0 \geq \pi\sqrt{d/a}$, then $h_\infty = \infty$.*

Proof. If we assume $h_\infty < \infty$, Theorem 6 implies $h_\infty \leq \pi\sqrt{d/a}$. However, it follows from $h'(t) > 0$ and $h_0 \geq \pi\sqrt{d/a}$ that h_∞ must satisfy $h_\infty > \pi\sqrt{d/a}$. This is a contradiction. □

Proposition 4. *Suppose $h_0 < \pi\sqrt{d/a}$ and define*

$$\overline{\mu} = \max\left\{1, \frac{b}{a}\|u_0\|_{C[0,h_0]}\right\} d\left(\frac{\pi^2 d}{a} - h_0^2\right)\left(2\int_0^{h_0} x u_0(x)dx\right)^{-1}.$$

If $\mu \geq \overline{\mu}$, *then* $h_\infty = \infty$.

The following theorem shows the existence of a threshold number $\mu^* \geq 0$, which separates the spreading and vanishing in case $h_0 < \pi\sqrt{d/a}$.

Theorem 9. *Assume* $h_0 < \pi\sqrt{d/a}$. *Then there exists a constant* $\mu^* \in [0, \overline{\mu})$ *depending on* u_0 *and* h_0 *such that, if* $\mu \leq \mu^*$, *then* $h_\infty \leq \pi\sqrt{d/a}$, *while if* $\mu > \mu^*$, *then* $h_\infty = \infty$. *Here* $\overline{\mu}$ *is a positive number given in Proposition 4. Moreover, if* $h_0 < (\pi/2)\sqrt{d/a}$, *then* $\mu^* > 0$.

2.4 Application II : Asymptotic Behaviors in the Bistable Type

We will apply the preceding results to the following free boundary problem with a bistable reaction term:

$$\begin{cases} u_t = du_{xx} + u(u-c)(1-u), & t > 0,\ 0 < x < h(t), \\ u(t,0) = u(t,h(t)) = 0, & t > 0, \\ h'(t) = -\mu u_x(t,h(t)), & t > 0, \\ h(0) = h_0,\ u(0,x) = u_0(x), & 0 \leq x \leq h_0, \end{cases} \tag{2.8}$$

where μ, h_0 and d are positive constants and c is a constant satisfying $0 < c < 1/2$. In addition, (u_0, h_0) is assumed to satisfy (1.3) and (1.4).

To study the asymptotic behavior of solutions for (2.8), we will prepare some results on auxiliary fixed boundary problems. Let l be any positive number and let φ be a nonnegative continuous function such that $\varphi \not\equiv 0$. We consider

$$\begin{cases} u_t = du_{xx} + u(u-c)(1-u), & t > 0,\ 0 < x < l, \\ u(t,0) = u(t,l) = 0, & t > 0, \\ u(0,x) = \varphi(x), & 0 < x < l \end{cases} \tag{2.9}$$

and the related stationary problem:

$$\begin{cases} dq'' + q(q-c)(1-q) = 0, & 0 < x < l, \\ q(0) = q(l) = 0, \\ q(x) \geq 0, \quad 0 < x < l. \end{cases} \tag{2.10}$$

Here it should be noted that (2.9) is a gradient system; so that any solution $u(t,x)$ of (2.9) converges to a stationary solution of (2.10) as $t \to \infty$ (see Brunovsky and Chow [1] and Hale and Massatt [6]).

As to the structure of solutions to (2.10), we have the following results.

Proposition 5. *There exists a positive number* L *with the following properties.*

(i) *If* $l < L$, *then* $q \equiv 0$ *is a unique solution of* (2.10);

(ii) *If* $l = L$, *then there exists a unique positive solution* $q(x; L)$ *of* (2.10);

(iii) *If* $l > L$, *then* (2.10) *has two positive solutions* $q_1(x; l)$ *and* $q_2(x; l)$ *such that* $q_1(x; l) < q_2(x; l)$ *in* $(0, l)$.

For the proof of this proposition, see Smoller [13, Theorem 24.13] or Smoller and Wasserman [14].

The stability of $q_i(x; l)$ $(i = 1, 2)$ with $l > L$ can be stated as follows.

Proposition 6. *For* $l > L$, *let* $u = u(t, x)$ *be the solution of* (2.9). *If* φ *satisfies* $\varphi(x) > q_1(x; l)$ (*resp.* $0 < \varphi(x) < q_1(x; l)$) *in* $(0, l)$, *then*

$$\lim_{t \to \infty} u(t, x) = q_2(x; l) \ (\textit{resp. } 0) \quad \textit{uniformly in } [0, l].$$

For the proof, see Matano [9] and Smoller [13].

We will study spreading and vanishing in the bistable case. Applying Corollary 1, we can prove the following theorem on spreading.

Theorem 10. *Suppose* $h_0 > L$ *and* $u_0(x) \geq q_1(x; h_0)$ *in* $[0, h_0]$. *Then the solution* (u, h) *of* (2.8) *satisfies* $h_\infty = \infty$ *and*

$$\lim_{t \to \infty} u(t, x) = v_B(x) \ \textit{uniformly in any compact subset of } [0, \infty),$$

where v_B *is a unique positive solution of*

$$\begin{cases} dv'' + v(v - c)(1 - v) = 0, & x > 0, \\ v(0) = 0. \end{cases} \tag{2.11}$$

The following proposition is useful to construct an upper solution of (2.8).

Proposition 7. *Define* $F(u) = \int_0^u s(s - c)(1 - s)\, ds$ *and* $a^* = \min\{a > 0;\ F(a) = 0\}$. *Then*

$$\begin{cases} dw'' + w(w - c)(1 - w) = 0 & \textit{in } \mathbb{R}, \\ w(0) = a^*, \ w'(0) = 0 \end{cases} \tag{2.12}$$

has a unique solution $w(x)$. *Moreover,* $w(x)$ *is monotone decreasing* (*resp. monotone increasing*) *for* $x \geq 0$ (*resp.* $x \leq 0$) *with* $w(x) = w(-x)$ *and* $\lim_{x \to \pm\infty} w(x) = 0$.

We now prove the vanishing property in the bistable case.

Theorem 11. *Let* $0 < h_0 \leq \infty$ *and assume* $0 \leq u_0(x) \leq w(x - x_0)$ *for* $x \in (0, h_0)$ *with some* $x_0 \in \mathbb{R}$, *where* $w(x)$ *is the unique solution of* (2.12). *Then the solution* (u, h) *of* (2.8) *satisfies*

$$\lim_{t \to \infty} \|u(t, \cdot)\|_{C[0, h(t)]} = 0.$$

Remark 3. *The assertions of Proposition 7 and Theorem 11 also hold true if the bistable term is replaced by a more general function* $uf(u)$ *in* (FBP) *which behaves like a bistable term. For general function* f, *define* $F(u) = \int_0^u sf(s)\,ds$ *and* $a^* = \min\{a > 0;\ F(a) = 0\}$. *Then* F *takes its local maximum at* 0 *and* F' *is positive at* a^*; *so we can construct a unique positive solution of*

$$\begin{cases} dw'' + wf(w) = 0 & \text{in } \mathbb{R}, \\ w(0) = a^*,\ w'(0) = 0 \end{cases}$$

with the same properties as w *in Proposition 7.*

In the bistable case, vanishing of type (i) (of (II)) can happen if h_0 and μ are sufficiently small.

Proposition 8. *Let* (u, h) *be any solution of* (2.8) *with* $h_0 < \pi\sqrt{d/(1-c)}$. *There exists* $\mu^* \geq 0$ *depending on* u_0 *and* h_0 *such that, if* $\mu \leq \mu^*$, *then* $h_\infty \leq \pi\sqrt{d/(1-c)}$ *and* $\lim_{t\to\infty} \|u(t,\cdot)\|_{C[0,h(t)]} = 0$.

3 Summary

In this section, we will consider our results from a biological point of view. The results implies that

- if the free boundary (spreading front of species) stays in a bounded interval, then the species must vanish eventually.
- if initial population density is larger than the solution of elliptic problem (2.2) in initial habitat, then the spreading is successful in the whole region $(0, \infty)$. The eventual distribution of the species obeys the solution of the elliptic problem (2.3).

More detailed information is obtained when we take an individual nonlinearity. For example, in the logistic case, spreading always occurs regardless of the initial population size in case $h_0 \geq \pi\sqrt{d/a}$. In the bistable case, even if h_0 is large, the vanishing occurs for small u_0. This is due to "Allee effect" of bistable nonlinearity, which means that the growth rate is negative for small population density.

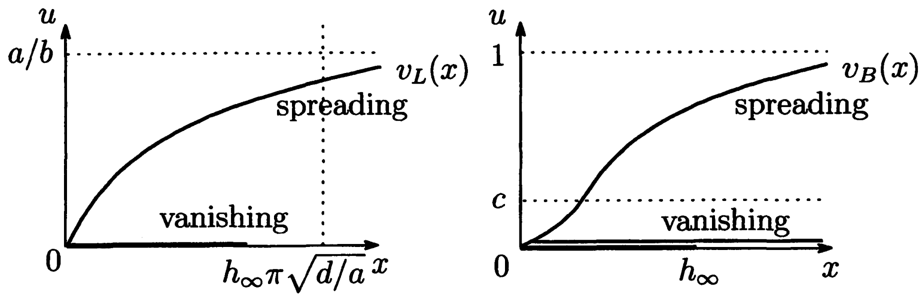

Figure 1 : Asymptotic Behaviors (logistic type, bistable type)

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